Performance Based Learning and Assessment Task

Tiling a Kitchen Floor

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
Students will use a variety of geometrical concepts and skills as they work with a partner to create a model representing the number of tiles that would be needed to cover a kitchen floor and calculate the costs of the tile. Students will also calculate the amount of potential waste involved and the estimated costs.

II. UNIT AUTHOR:
Yolonda Shields, Benjamin Franklin Middle School, Franklin County

III. COURSE:
Geometry

IV. CONTENT STRAND:
Measurement and Geometry

V. OBJECTIVES:
Students will be able to: 1) solve real-world problems involving angles of polygons. 2) identify tessellations in construction and nature. 3) use similar geometric objects in two-dimensions to solve real-world problems about similar geometric objects (making a scale drawing to model the kitchen floor). 4) solve real-world problems using right triangle trigonometry and properties of right triangles (finding the area of regular hexagons). 5) determine the total estimated costs for covering a kitchen floor with tile.

VI. REFERENCE/RESOURCE MATERIALS:
Students will need: TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, Internet, Tessellation Blocks (optional), GeoGebra or Geometer's Sketchpad Software (optional), Ruler, Tape, Scissors, Protractor, Assessment Rubric, Copy of Performance Task, Word Processing Software (i.e. Microsoft Word or Google Docs), Copy of Benchmarks, Copy of Hexagon Template(for those who may need this accommodation)

VII. PRIMARY ASSESSMENT STRATEGIES:
The assessment list for each of the activities will contain all the essential components for this mathematics activity. This includes the mathematics content, process skills, and requirements for the finished product. There is also an option of providing a copy of the attached hexagon template to students in need.

VIII. EVALUATION CRITERIA:
Students will be evaluated on their successful completion of the activity and by their final product.

IX. INSTRUCTIONAL TIME:
Two ninety-minute class sessions

Tiling a Kitchen Floor

Strand
Measurement & Geometry

Mathematical Objective(s)
Students will be able to: 1) solve real-world problems involving angles of polygons. 2) identify tessellations in construction and nature. 3) use similar geometric objects in two-dimensions to solve real-world problems about similar geometric objects (making a scale drawing to model the kitchen floor). 4) solve real-world problems using right triangle trigonometry and properties of right triangles (finding the area of regular hexagons). 5) determine the total estimated costs for covering a kitchen floor with tile.

Related SOLs
SOL G.10
SOL G.3
SOL G.14
SOL G.8

NCTM Standards
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest.
- Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Draw and construct representations of two-dimensional geometric objects using a variety of tools
- Make decisions about units and scales that are appropriate for problem situations involving measurement
- Understand and use formulas for the area of geometric figures.
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements
- Solve problems involving scale factors, using ratio and proportion
- Organize their mathematical thinking through discussion with peers
- Communicate their thinking clearly to teacher and peers

- Analyze and evaluate the mathematical thinking and strategies of their partners
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and apply mathematics in contexts outside of mathematics
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Create and use representations to record and communicate mathematical ideas
- Select, apply, and translate among mathematical representations
- Use representations to model and interpret physical and mathematical phenomenon

Additional Objectives for Student Learning

Theatre Arts I: Introduction to Theatre

- TI. 4 (apply the creative process in storytelling, playwriting, and acting by creating and writing a monologue and/or scene)
- TI. 1 (explore theatre as an ensemble art through group interaction by developing communicating strategies, collaborating to implement personal artistic choices, and respecting the ideas and viewpoints of others)

Materials/Resources

Students will need: TI-83 Plus (or higher) Graphing Calculator, Computer, Pencil, Paper, Internet, Tessellation Blocks (optional), GeoGebra or Geometer's Sketchpad Software (optional), Ruler, Tape, Scissors, Protractor, Assessment Rubric, Copy of Performance Task, Word Processing Software (i.e Microsoft Word or Google Docs), Copy of Benchmarks, Copy of Hexagon Template(for those who may need this accommodation)

Assumption of Prior Knowledge

- Definition of tessellation and ability to connect this to "covering the kitchen floor with tile."
- Knowledge of which polygons will tessellate the plane using knowledge of interior angles
- How to determine the total cost of an item based on given information
- Finding the area of regular polygons (including hexagons using right triangle trigonometry)
- Some experience with graphing calculator technology and Geometer's Sketchpad / GeoGebra Software; Experience with using Word Processing Software
- Creating models and using proportional reasoning
- Understanding of area and perimeter
- Familiarity with scaled drawings and blueprints (kitchen floor plans)

Introduction: Setting Up the Mathematical Task

Students will use mathematics, logical thinking, research, and their knowledge of perimeter, area, polygon angles, and tessellations to come up with solutions for the kitchen tiling problem they are presented by a neighbor. The mathematical goal of this activity is for students to choose between a hexagon and an octagon shape to use to cover the surface of a neighbor's

kitchen floor with tile. The students will have to determine which of these regular shapes will tessellate the kitchen floor. In doing so, consideration of the amount of waste (if any) must be taken as the students use a variety of formulas to determine the number of hexagon tiles necessary to cover the floor without any gaps or overlaps. Once this number is determined, students will have to determine the estimated costs of the project, and create a model showing how many tiles would be needed. This information will be presented to the class in any desired format (i.e. Skit that is roughly 3 minutes long). The students will have to do research to find the tile they plan to use for the floor, keeping in mind that all of the sides have to be congruent. The students will use a copy of the kitchen blueprint as a guide. The goal is also to help the students use their knowledge of Geometry and other mathematical concepts to help their neighbor make an informed decision in her kitchen tiling project. Students will have the opportunity to use the Internet, word processing software, and GeoGebra or Geometer's Sketchpad to research, type out key information, and create a hexagon or octagon template that will be useful during the presentation and for creating a model of the kitchen floor covered with the appropriate tile pieces. Students will gain valuable skills as they gather information and communicate their mathematical ideas.

- Discuss the activities for the day (refer to displayed Agenda)
- Motivating activity to introduce the goal of the task/activity
    - Inform students that their valuable mathematical input is needed to help a neighbor design her kitchen floor and cover the floor with tile. They will have to choose between 2 different options/shapes, show evidence why, determine estimated costs by figuring out how many tiles would be needed to cover the floor and researching the costs per tile, create a model of how the floor would look, and then present this information in a 3-minute presentation in any format of their choosing (i.e. Skit). Inform students that they will need to consider any potential waste involved with this project.
- Distribute the task and assessment rubrics to the students; Teacher will give an outline of the performance assessment task and the timeframe for completion as students look at the typed version that is passed out to them.
- Students can pick a partner or the teacher can create the groups
- In their groups of 2, students will begin brainstorming steps for addressing this real-world task
- Teacher will act as facilitator and will ask questions or give prompts to the students, such as things they should consider. The teacher will reinforce the idea that he or she is going to be in this role and that the students will be responsible for developing a plan and implementing that plan to come up with a solution.
- Students will be asked to draw upon their prior knowledge to come up with solutions

- Teacher will help students understand the task by effectively answering their questions if they arise.
- Students will have to use Mathematics to solve the problem, but will be given access to a computer and the Internet to help them.
- To make the students' mathematical thinking and understanding public, students will collaborate with their partners, come up with a solution or solutions, and share their ideas through a culminating activity that involves the presentation of their information.

Student Exploration

- Students will first need to analyze the kitchen floor plan to determine the dimensions of the area that needs to be covered. Students will also need to determine if each of the 2 shapes (hexagons and octagons) will indeed tessellate the plane and what would be the disadvantage if we used a shape that does not tessellate the plane (i.e gaps/more waste). As a visual, students could use tessellation blocks to demonstrate. Students will also be determining how many tiles would be needed to cover the kitchen floor and the total cost. As part of the presentation, students will create an accurate model demonstrating how the floor would look after it is covered (taking its dimensions and the dimensions of the tile into consideration). Students will have access to the computer for research, typing their paper, and using GeoGebra/Geometer's Sketchpad Software if desired. They will also have access to the Internet and a graphing calculator to help them with these calculations. Students with accommodations may be given a copy of the Hexagon Shape Template to use (see attached resource at the end of this task).
- The teacher will be a facilitator who helps the kids stay on task. He or she will help guide the students and use a variety of questions to help guide the students as they participate in their exploration.

Monitoring Student Responses

- I expect students to work together as they communicate their thinking and their new knowledge with each other and with the teacher.
- The teacher will help students with any clarification needs and assist students who are having difficulties by helping them connect their previous experiences to these new ones.
- If there are students who are ready to move forward, differentiation will be used and the students can explore additional ways to solve the problem, additional topics (such as using multiple shapes), additional costs associated with tiling the kitchen floor (i.e. the adhesive/tools needed to secure the tiles), etc. This information could also be included in the final product.
- Students who are ready to prepare their class presentation, can go ahead and move to the computer to begin typing out key ideas/information (they will earn bonus points).

- Closure will involve the students sharing their results through a culminating activity, such as a presentation to the class. This will be followed by a group discussion and with the teacher providing feedback.

Assessment List and Benchmarks

Student handouts, rubrics, and benchmarks are attached.

Performance Based Assessment Task

Name(s): _____

_____

Date: _____

Geometry Teacher: _____

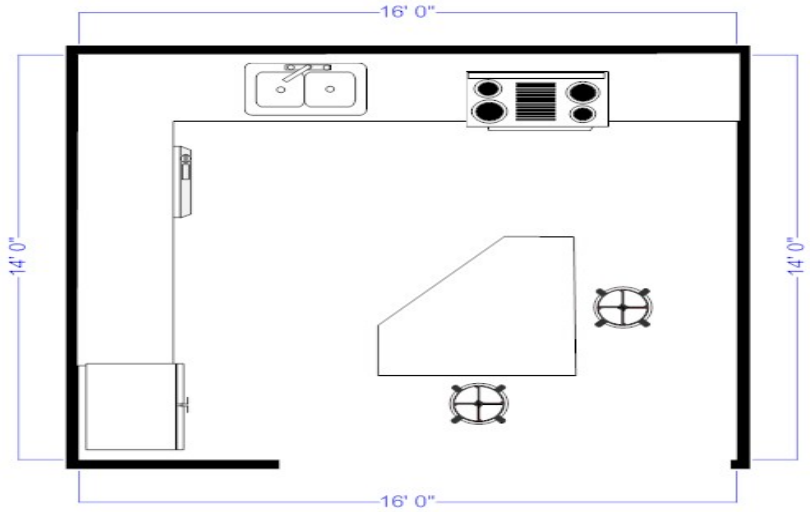
Tiling a Kitchen Floor

Background Information

You and your partner are being asked for your mathematical input. A neighbor is planning on tiling her kitchen floor and remembers hearing someone tell her years ago that there are only certain shapes that can be used by themselves to tile a kitchen floor and that it had something to do with Math. So, since she knows that you both are the brightest Math students she knows, she comes to you! She asks you to choose the shape for her, estimate the cost of the tile, and show her a picture of how it would look.

Your Neighbor's Requirements

She hands you a blueprint of her kitchen (pictured below) and tells you that the only requirements that she has is that no matter what shape is selected, it has to be the same shape and the same size throughout the kitchen. She also tells you that she does not mind if you have to cut the shape into smaller pieces to fill in any gaps along the edges if needed. You and your partner agree that your favorite polygons are hexagons and octagons and that you will not use squares, rectangles, or triangles as the main shape, but need to use your mathematical knowledge to see if they both work, one of them works, or none of the shapes work.



* The counter spaces are 3 feet wide in this L-Shaped Kitchen

Picture source:

http://4.bp.blogspot.com/-_rVH412ajA0/TOIJNal_pI/AAAAAAA

AAQs/3fkw3hR92xc/s1600/Island_Kitchen_Floor_Plan_L.jpg

Determining the Best Option & Costs

As current Geometry students you are more than equipped to handle and be successful at this authentic challenge! You must show your work and be able to explain your steps for solving this problem. Keep in mind that Mathematics and research can help you determine the best option and the associated costs of using either hexagons or octagons. You and your partner must demonstrate that you both worked together to come up with a plan for choosing your option and the teacher will be making sure that you are actively participating in discussions at least 3 times.

Class Presentation

Be prepared to present your information to the class in any way you desire (i.e. – a skit acting out what your conversation would be like with your neighbor as you share your mathematical findings –about 3 minutes). Make sure the presentation is organized and be prepared to answer any questions that are posed to you by the teacher and your classmates. If you type the key concepts / information used for your presentation, you will earn bonus points!

			Earned Assessment	
#	**Element**	**Point Value**	**Self**	**Teacher**
1	Student participated in discussions with his/her partner and the class by speaking or asking a question at least three times (recorded with an observation checklist by instructor)	2		
2	Student developed a plan for determining which shape(s) they could use to tile the kitchen floor (hexagons and/or octagons)	2		
3	Student documented any research done and the information gathered to address this real-world problem	2		
4	Student recorded measurement findings while completing the activity and used proper mathematics to verify solutions	2		
5	Student shows work, with correct answers, and explanations while using correct formulas for this kitchen tiling project	2		
6	Student correctly identified a shape that could cover the kitchen by itself (hexagon / octagon)	2		
7	Student explained their mathematical reasoning behind their shape selection	2		
8	Student used the dimensions of the kitchen appropriately when being substituted into the selected formulas	2		
#	**Element**	**Point Value**	**Self**	**Teacher**

9	Student can show why their formulas work for determining how many tiles would be needed to cover the kitchen floor	2		
10	Student took the amount of waste (if any) involved with using their shape into consideration	2		
11	Student determines the total estimated costs associated with using their shape to cover the kitchen floor, using a reasonable price obtained from a verified and documented source (i.e. Price Quote from the Lowes Home Improvement Hardware Store website)	2		
12	Student creates an accurate model of how the kitchen floor would look after being tiled	2		
13	Student demonstrates use of technology (i.e. Internet, GeoGebra, Microsoft Word) in their solutions or final products	2		
14	Student actively participates in a roughly 3 minute class presentation by both offering information and responding to the higher-order questions posed to him or her by the teacher and classmates after the presentation	2		
15	Student's work and presentation is well-organized	2		
16	Student's work is neat	2		
17	Student uses Word Processing Software to type presentation key ideas / information (**Bonus Points!)**	2		
	Total (Out of 32)			
	Rubric for Activity 2			

#	Element	0	1	2
1	Student participated in discussions with his/her partner and the class by speaking or asking a question at least three times (recorded with an observation checklist by instructor)	Student did not speak or ask any questions	Student contributed 1 or 2 times	Student contributed 3 or more times
2	Student developed a plan for determining which shape(s) they could use to tile the kitchen floor (hexagons and/or octagons)	No evidence of a plan provided	Evidence of plans for only one of the 2 shapes	Evidence of plans for both shapes
3	Student documented any research done and the information gathered to address this real-world problem	No documentation	Minimal documentation	Sufficient documentation
4	Student recorded measurement findings while completing the activity and used proper mathematics to verify solutions	Measurements are not provided	Some measurements are provided	All measurements are provided
5	Student shows work, with correct answers, and explanations while using correct formulas for this kitchen tiling project	Insufficient	Work is shown, incorrect answers, and reasoning explained	Work is shown, with correct answers, and clear explanations
6	Student correctly identified a shape that could cover the kitchen by itself (hexagon / octagon)	Does not identify a shape	Identifies a shape, but it is incorrect	Correctly identifies a shape
7	Student explained their mathematical reasoning behind their shape selection	Does not explain	Attempts to explain, but does so incorrectly	Clearly explains reasoning
8	Student used the dimensions of the kitchen appropriately when being substituted into the selected formulas	No indication of appropriate use of substitution into these formulas	Some dimensions are substituted correctly into these formulas	All dimensions are substituted correctly into these formulas
#	**Element**	**Point Value**	**Self**	**Teacher**

9	Student can show why their formulas work for determining how many tiles would be needed to cover the kitchen floor	Cannot show how the formulas work	Partially shows why the formulas work	Correctly and completely shows how the formulas work
10	Student took the amount of waste (if any) involved with using their shape into consideration	No consideration for the amount of waste involved	Understands and explains why there could be some waste involved, but unable to use math to show how much	Understands the amount of waste involved (if any) and uses math to correctly show much would be involved
11	Student determines the total estimated costs associated with using their shape to cover the kitchen floor, using a reasonable price obtained from a verified and documented source (i.e. Price Quote from the Lowes Home Improvement Hardware Store website)	No evidence of cost calculations or use of a verified and documented source	Missing one component: cost calculations or use of a verified and documented source	Both components are present and accurately used
12	Student creates an accurate model of how the kitchen floor would look after being tiled	No model created	Model created, but inaccurate	Accurate model created
13	Student demonstrates use of technology (i.e. Internet, GeoGebra, Microsoft Word) in their solutions or final products	No evidence of technology use	Technology use is present, but is not effectively used	Technology use is evident in the solution or final product and is effectively used
14	Student actively participates in a roughly 3 minute class presentation by both offering information and responding to the higher-order questions posed to him or her by the teacher and classmates after the presentation	Student does not participate and no follow-up questions are answered	Student passively participates and follow-up questions are answered, but are not correct or clear	Student actively participates and follow-up questions are answered logically and correctly (or close to correct)
15	Student's work and presentation is well-organized	No evidence of organization	Not fully organized	Well-organized work and presentation
#	**Element**	**Point Value**	**Self**	**Teacher**

16	Student's work is neat	Lacks neatness	Needs improvement	Neat and legible
17	Student uses Word Processing Software to type presentation key ideas / information (**Bonus Points!)**	Did not use this software	Did not use this software	Used this software

Benchmark

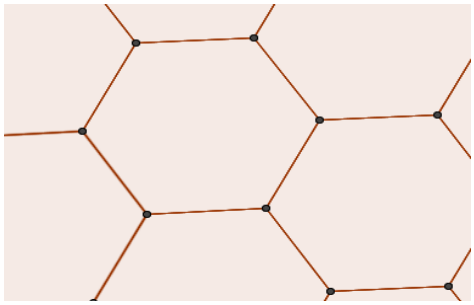
This is how we looked at this authentic problem:

1. Since all the sides of the tiles have to be the same length, these polygons are regular polygons.
2. When you are tiling a kitchen floor, it is important to make sure that there are no gaps or overlaps. This means that these tile pieces must tessellate the floor since it is like a plane in Mathematics. In order to determine which shape we want to use (hexagon or octagon), we must first find the interior angle of each of these regular polygons:
    a. The interior angle of a regular polygon can be calculated by the following formula: (n-2)180 /n, where n is the number of sides
    b. Then, we must see if the interior angle measures of the polygon is a factor of 360. In other words, the interior angles must divide into 360 with nothing left over (no remainder). This is a requirement!
3. Once we figure out which shape we will use, we will then do research on the internet to look up these tiles, choose one we would like to use, and find the associated costs for each tile.
4. Next, we will look at the kitchen blueprint and find the dimensions of the actual floor that we are covering.
5. Then, we will find the area of this floor and consider its perimeter when determining how many tiles will be needed to cover the kitchen floor and the amount of waste involved.
6. Once we find this, we will make a model to show our neighbor how the floor would look.
7. Then, we will use the costs associated with each tile and the total number of tile needed to find the total cost of the tile material.

Here is our work associated with each of the steps above (starting at #2):

2.
    a. Interior Angle Measures of Regular Hexagons and Regular Octagons:
        i. Work for Hexagon:
            1. (6-2)180 / 6 = 720/6 = 120 degrees
        ii. Work for Octagon:
            1. (8-2) 180 / 8 = 1080 / 8 = 135 degrees

b. Factor of 360?
    i. Work for Hexagon:
        1. 360 / 120 = 3 This has no remainder, so it works!
        2. Regular hexagons have interior angles that are 120 degrees each. Since 120 is a factor of 360, they will also tessellate the plane. Around each vertex, there would be 3 regular hexagons.
    ii. Work for Octagon:
        1. 360 / 135 = 2 r 90 It has a remainder of 90, so it does not work! There would need to another shape w/ a 90 degree angle to fill this gap, like a square.
    iii. Conclusion: So, we must use hexagons! This is how it would look around each vertex:



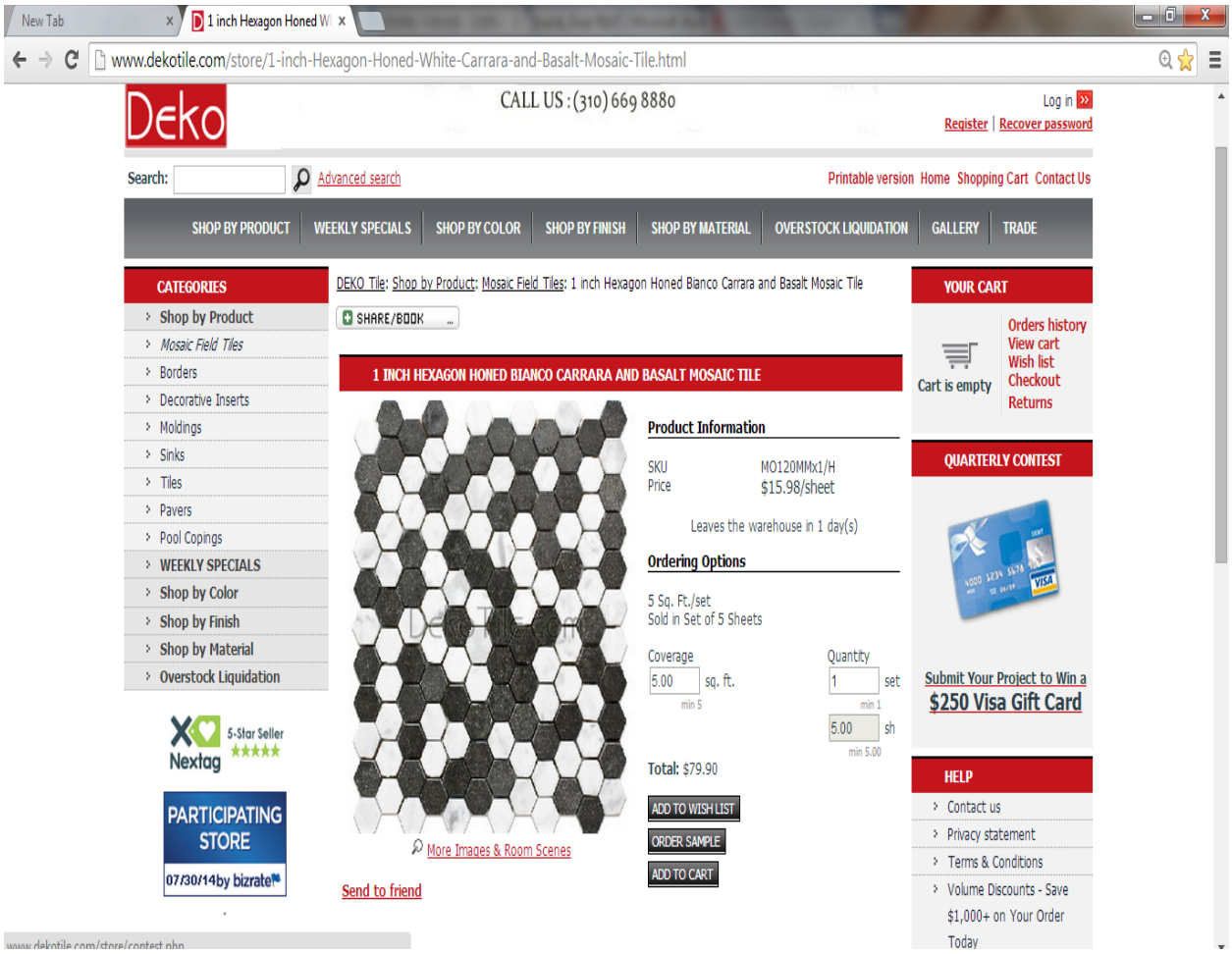
3. Internet research we found on the hexagonal tiles and their costs:

* http://www.dekotile.com/store/1-inch-Hexagon-Honed-White-Carrara-and-Basalt-Mosaic-Tile.html

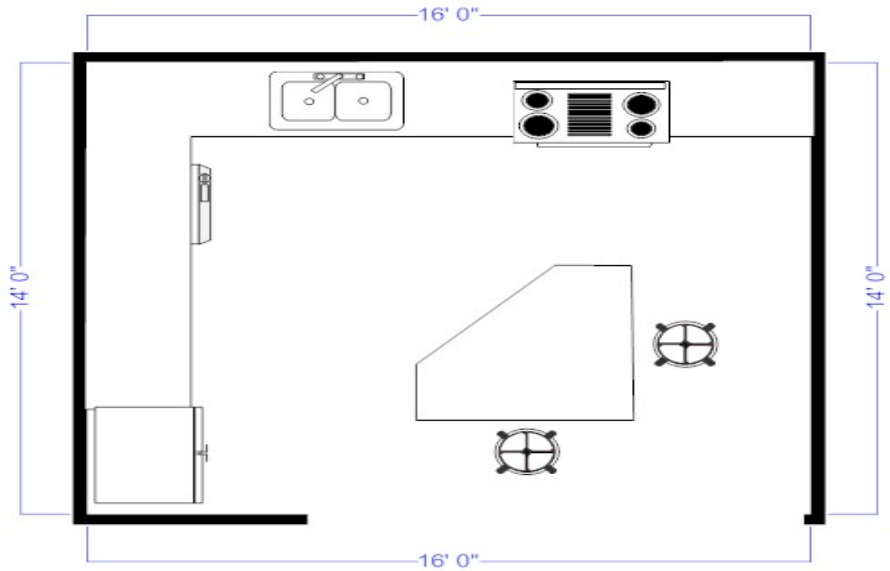
* We chose this tile (pictured below) because we liked the design and the materials seem to be of good quality:

* These 1" Hexagon Honed Bianco Carrara and Basalt Mosaic Tiles come in a 12" x 12" mesh and each sheet covers 5 square feet.

* It costs $15.98 per sheet and it comes in boxes of 5. Therefore, it has to be bought in increments of 5. So, each box will cover 25 square feet and will cost $15.98 * 5 = $79.90.



4. Blueprint Revisit & Actual Dimensions



* The counter spaces are 3 feet wide in this L-Shaped Kitchen & the Island in the middle of the floor is removed so that the floor is bare.

* Actual Dimensions of Kitchen Floor is (14-3)ft. ft. x (16–3)ft. or 11 ft. x 13 ft.

5. Area of the kitchen floor is 11 x 13 = 143 ft^2 or 143 square feet.

   How many boxes of tile do we need?

   Since each box covers 25 square feet and the area of the kitchen floor is 143 square feet, we would need 6 boxes since 143 / 25 = 5.72 boxes.

   How much waste is involved?

   Not much waste is involved. We would have approximately 7 square feet extra since 150 – 143 = 7. It is possible that this will cover any mistakes that may occur during the placement of the tile.

6. Model to show neighbor:

   This is a zoomed in picture of how the tile looks:

7. Finding the total cost of the tile:

   Work: Since each box of tile costs $79.90 and we need 6 boxes, we multiplied each of these values together to get the total estimated tile cost, which is $479.40.

As optional and supplemental material that is not a part of this performance based assessment:

Challenge: Give students a more complicated kitchen floor to tile: ones with a variety of shapes not just rectangular, ones with an island in the middle of the floor that can't be removed – you have to work around it, etc.

Additional kitchen floor plan ideas can be found at: http://damaris-daria.blogspot.com/2013/06/kitchen-floor-plan-layouts.html

Also, students can include additional costs such as the tiling glue, etc.